Alexander Christenson

EE109: Convex Optimization

Professor Mai Vu

Final Project – Problem Formulation

**Volume Forecasting for Multi-Period Trading via Convex Optimization**

**Abstract – Insert abstract here when done**

**Background**

In 1952 Harry Markowitz published his seminal paper “Portfolio selection” and gave birth to the field of portfolio theory. In the preceding years many developments have been built on his work, yielding increasing levels of insight into financial market functionality and financial decision making. The fundamental question answered by his paper is: How should an investor allocate funds among a possible set of asset choices?

To address this question Markowitz suggested two novel approaches:

1. A method to quantify the risk and return of an asset using statistics (variance, expected value)
2. Investors should consider risk and return together and determine the allocation of funds based off of a risk-return profile of their portfolio.

Before the introduction of his theory, investors used risk and return in a disjointed fashion, rather than treating them as a connected trade-off between one measurement and the other. This approach has provided great value to investors. By quantifying and treating risk and return as a single metric, investors could then evaluate a portfolio’s return based off of both the individual potential returns of an asset and the relationship of assets returns to each other, rather than viewing the returns of a portfolio as the weighted average of the individual returns.

A simple example of the pre and post portfolio theory approach:

Take a portfolio composed of oil, transportation, and industrial manufacturing securities. The pre portfolio theory approach would be analyzing the individual returns of the assets. Our qualitative analysis of the companies indicates that all three of the securities will increase in value, and when we quantify and project the financial growth, we find the companies are projected to grow by the same amount. In this case we would want to purchase only one of the securities to minimize the cost of purchasing the security (a broker is required to link a buyer and seller in the market – known as market makers, and they charge a fee per order for their service).

Using the methods of portfolio theory an investor would likely find a connection between the three companies. In this case, if the price of oil increases, the cost of transportation will increase and decrease the value or return of our transportation security. The rise of oil price will likely increase the value of our oil drilling component manufacturer and the share price will rise together. Our analysis of the individual securities implied that all three will grow, but the connection between the industries implies that their growths are linked to each other and our financial analysis is likely risky for each of them. In order to reduce the risk of the portfolio we would want to distribute roughly half our funds to the transportation company and half between the oil and oil component manufacturer. In this case we hedge our projections against each other and reduce the risk of an overall loss, while maximizing the return.

What portfolio theory allows us to do is extrapolate this approach to a wide variety of assets in a highly complicated and inter-connected system. This is especially useful where the connections between assets may not be as readily apparent, e.g. adding corn to your portfolio may reduce the risk associated with investing in an aerospace security because there may be an illogical correlation between the two. Applying this method across a portfolio allows investors to generate higher returns at a lower risk.

Markowitz formulated portfolio theory as an optimization problem as mean variance optimization (MVO). The MVO has an infinite number of potential portfolios to select from to generate a desired return, and then from there the risk is minimized. The formulation of this problem is in the appendix. To apply portfolio theory in practice investors will utilize quantitative trading algorithms. These algorithms have become increasingly popular over the past decade as the cost of computation has decreased. One of the problems to develop from portfolio MVO is that the model does not include the cost of transaction for each of the stocks.

To address this a series of new trading algorithms have been developed. Most of the work done in this field has been on approaching the problem from a single period point of view, referred to as Single Period Optimization (SPO). These SPO techniques give investors guidance as to which and how much of the funds should be distributed to a selection of assets, effectively performing the same analysis as MVO but adding additional information on the transaction and hold costs of assets. A limitation in the technique is it does not take into account the future returns of the portfolio. To address this a new field of Multi-Period Optimization (MPO) has been created to address this. It answers the question for investors of when each of the assets should be purchased and has become more viable as computation has become less costly.

**Assumptions**

One of the major assumptions made in the building body of work on multi-period trading is the completeness of order execution. In order to be able to purchase an asset there must be a buyer and a seller on the opposite side who can agree on a price determined by a market maker. Much research has been done on this is a single period setting. My objective in this project is to extend the single period market-making optimization techniques to a multi-period framework. The consequence of this would allow investors to account for trades that cannot be fully executed by their models.

For example:

Say you have a trade you want to execute for 1000 shares of company X at the market price of $10 per share. The multi-period trading model assumes you are able to purchase 1000 shares for $10 per share. You put the order in with your broker who tries to make a market and find 1000 shares for you to purchase at that price. Your broker is only able to find 500 shares at that price, but as you purchase the shares at that price, other sellers increase the value they are willing to sell for and are willing to sell for $11 per share. Your broker can find and execute the trade for an average price of $10.50 a share. You then own 1000 shares for $10,500 and have paid a $500 premium over the expected price, and the price your model anticipated you paying. This problem increases dramatically when you scale the price and volume of the assets to be purchased.

**Problem Formulation**

**Explain how they do it in the single period**

**Explain constraints**

**Do same for multi-period**

**Show it that they don’t consider the same complexity**

**Proposed problem – how you can incorporate the complexity**

**Show a new formulation**

**Want to see some sort of algorithm and some results**

**Propose a very simple algorithm and show how you can take multi-period algorithm and modify it a little bit**

**Run and see how it goes**

Overview of single and multi-period show the different problem formulations and algorithms

Simpler model

Show at least one or two of the algorithms

Come up with some kind of tweak

**Appendix**

Insert the MVO problem formulation here.